UNCERTAINTY CALIBRATION OF LARGE-ORDER MODELS OF BRIDGES USING AMBIENT VIBRATION MEASUREMENTS

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ABSTRACT

A computational efficient Bayesian inference framework based on stochastic simulation algorithms is presented for calibrating the parameters of large-order linear finite element (FE) models of bridges. The effectiveness of stochastic simulation tools to handle large-order linear models in Bayesian analysis is demonstrated by calibrating a high fidelity FE model of the Metsovo bridge with several hundreds of thousands of DOF, using experimentally identified modal frequencies and mode shapes based on ambient vibration measurements collected from a wireless mobile measuring system. The mode shapes of the bridge are assembled using the identified modal characteristics from a number of different sensor configurations, involving reference and moving sensors, optimally placed on the bridge deck to adequately cover the whole bridge span. The identified finite element models and their uncertainties are representative of the initial structural condition of the bridge and can be further used for structural health monitoring purposes.

KEYWORDS: *Structural dynamics, Modal estimation, Bayesian inference, Model updating, Bridges.*

INTRODUCTION

The evaluation of the actual dynamic characteristics of bridges, such as modal frequencies, modal damping ratios and mode shapes, through vibration measurements, as well as the development of high-fidelity finite element models (FE), has been attracting an increasing research effort worldwide. Measured response data of bridges mainly from ambient vibrations offer an opportunity to study quantitatively and qualitatively their dynamic behaviour. These vibration measurements can be used for estimating the modal characteristics of the bridges, as well for calibrating the corresponding FE models used to simulate their behaviour. The information for the identified FE models and their associated uncertainties are useful for checking design assumptions, for validating the assumptions used in model development, for improving modelling and exploring the adequacy of the different classes of FE models, for identifying possible soil-structure interaction effects, and for carrying out more accurate robust predictions of structural response. These models are representative of the initial structural condition of the bridge and can be further used for structural health monitoring purposes.

Bayesian methods for modal identification and structural model updating are used to develop high fidelity FE models of bridges using modal characteristics and their uncertainties identified from ambient vibration measurements. Recent developments in Bayesian methodologies [1] based on ambient vibration measurements are used for estimating the modal frequencies and mode shapes of structures and their uncertainties. Due to the large size of the bridges, the mode shapes of the structure are assembled from a number of sensor configurations that include optimally placed reference sensors as well as moving sensors. The modal properties and their uncertainties are then integrated within Bayesian model updating formulations to calibrate the parameters of large scale

FE models as well as their associated uncertainty. Existing Bayesian formulations are extended to include uncertainties in the modal characteristics that are identified by the Bayesian modal estimation methodology. For complex posterior distributions, stochastic simulation algorithms can be conveniently used to sample from these distributions for parameter estimation, model selection and uncertainty propagation purposes. This requires a large number of system re-analyses which can increase substantially the computational effort to excessive levels if one simulation for high-fidelity large-order models requires several minutes or even hours to complete. Fast and accurate component mode synthesis techniques, consistent with the FE model parameterization, are used to achieve drastic reductions in computational effort [2]. Further computational savings are achieved by adopting parallel computing algorithms such as the Transitional MCMC to efficiently distribute the computations in available multi-core CPUs [3].

The proposed framework is demonstrated by calibrating the FE model of the Metsovo Bridge. It is demonstrated that high performance computing and model reduction tools integrated within Bayesian tools can be used to calibrate the uncertainty in these models within very reasonable computational time, despite the very large number of DOF of the order of hundreds of thousands or even several million.

1 FORMULATION FOR MODEL UNCERTAINTY CALIBRATION

The Bayesian methodology is used to calibrate the parameters of a FE model and estimate the uncertainties in these parameters using vibration measurements. According to the methodology, if $\underline{\theta}$ denotes the parameters of the FE model, the uncertainty in the parameters given measured data D is quantified by the posterior distribution that is obtained from Bayes theorem as

$$p(\underline{\theta} \mid D) = \frac{p(D \mid \underline{\theta})\ \pi(\underline{\theta})}{p(D)} \tag{1}$$

where $p(D \mid \underline{\theta})$ is the likelihood, $\pi(\underline{\theta})$ is the prior distribution of the uncertain parameters and $p(D)$ is the evidence of the finite element model.

To apply the Bayesian formulation for parameter estimation of linear FE models, we consider that the data consist of modal frequencies $\hat{\omega}_r$ and mode shapes $\underline{\hat{\phi}}_r \in R^{N_{0,r}}$, $r = 1,\ldots,m$, experimentally estimated using ambient vibration measurements, where m is the number of identified modes and $N_{0,r}$ the number of measured components for mode r. There are a number of techniques for estimating the modal frequencies and mode shapes from output only vibration measurements. Herein we use the Bayesian modal parameter estimation method proposed in [1]. In addition to the most probable values of the modal frequencies, the uncertainty in the modal frequencies is also quantified by Gaussian distributions and the standard deviation of such distributions for all identified modal frequencies is estimated.

The likelihood is built up using the following model prediction error for the modal frequencies and mode shapes. The prediction error equation for the r-th modal frequency is

$$\hat{\omega}_r^2 = \omega_r^2(\underline{\theta}) + e_r^{(e)} + e_r^{(m)} \tag{2}$$

where $\omega_r(\underline{\theta})$ is the model predictions of the r-th modal frequency for a particular value of the parameter set $\underline{\theta}$, $e_r^{(e)}$ is the experimental error taken to be Gaussian error of zero mean and standard deviation $\sigma_{e,r}$ identified from the modal identification technique, $e_r^{(m)}$ is the model error assumed to be Gaussian with zero mean and standard deviation $\sigma_{\omega,r}\hat{\omega}_r^2$, and $\sigma_{\omega,r} = \sigma_\omega$ is the model prediction error parameter for the modal frequencies, assumed herein to be the same for all modes. Following the formulation in [4], which avoids the use of mode correspondence, the prediction error equation for the r-th mode shape is

$$\underline{\hat{\phi}}_r = \underline{\varphi}_r(\underline{\theta}) + \underline{\varepsilon}_r^{(e)} + \underline{\varepsilon}_r^{(m)} \tag{3}$$

where $\underline{\varphi}_r(\underline{\theta}) = L_r\Phi(\underline{\theta})\underline{a}_r = \Phi_r(\underline{\theta})\underline{a}_r$ is the model predictions of the r-th mode shape, $\Phi(\underline{\theta})$ is the matrix of model mode shapes $\underline{\phi}_r(\underline{\theta})$, $r = 1,\ldots,m$ predicted from a particular value of the parameter set $\underline{\theta}$, $\underline{\varepsilon}_r^{(e)}$ is the experimental error taken to be Gaussian of zero mean and covariance matrix identified from the modal identification technique, $\underline{\varepsilon}_r^{(m)}$ the model error assumed to be Gaussian with zero mean and covariance matrix $\sigma_{\phi,r}^2 \| \underline{\hat{\phi}}_r \|^2 I$, and $\sigma_{\phi,r} = \sigma_\phi$ is the model prediction error parameter for the mode shapes, assumed herein to be the same for each mode. The vector $\underline{a}_r \equiv \underline{a}_r(\underline{\theta}) = [\Phi_r^T(\underline{\theta})\Phi_r(\underline{\theta})]^{-1}\Phi_r^T(\underline{\theta})\underline{\hat{\phi}}_r$ is chosen to guarantee that $\underline{\varphi}_r(\underline{\theta})$ is closest to $\underline{\hat{\phi}}_r$. The model prediction error parameters σ_ω and σ_ϕ are considered to be unknown and are incorporated in the unknown parameter set $\underline{\theta}$ in the Bayesian formulation.

Given (2) and (3) one readily derives that the likelihood function required in (1) is given by

$$p(D \mid \underline{\theta}) \propto \frac{1}{\sigma_\omega^m \sigma_\phi^M} \exp\left[-\frac{1}{2\sigma_\omega^2} J_\omega(\underline{\theta}) - \frac{1}{2\sigma_\phi^2} J_\phi(\underline{\theta}) \right] \tag{4}$$

where

$$J_\omega(\underline{\theta}) = \sum_{r=}^{m} \frac{a_r^T(\underline{\theta})\Lambda(\underline{\theta})a_r(\underline{\theta}) - \hat{\omega}_r^2 a_r^T(\underline{\theta})a_r(\underline{\theta})}{\hat{\omega}_r^2 a_r^T(\underline{\theta})a_r(\underline{\theta})}, \qquad J_\phi(\underline{\theta}) = \sum_{r=}^{m} \frac{\| \Phi_r(\underline{\theta})a_r(\underline{\theta}) - \underline{\hat{\phi}}_r \|^2}{\| \underline{\hat{\phi}}_r \|^2} \tag{5}$$

$\Lambda(\underline{\theta}) = diag[\omega_r^2(\underline{\theta})]$ and $M = \sum_{r=1}^{m} N_{0,r}$. The formulation is quite flexible to account for different number of measured components per mode shape, arising usually from the mode shape assembling process, as well as to avoid the mode correspondence which is often a non-trivial task in model updating formulations.

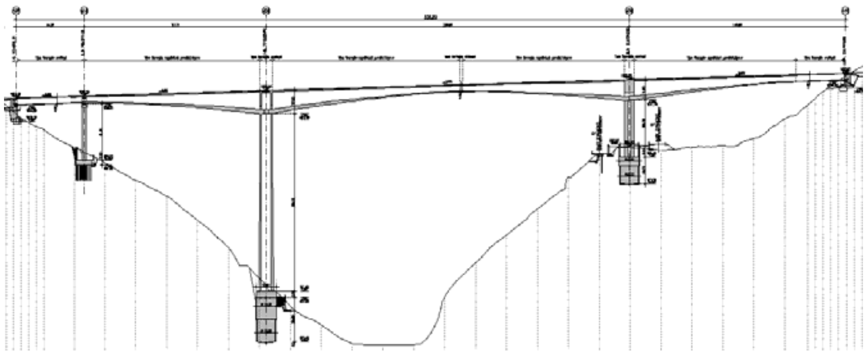

Figure 1: Longitudinal view of the Metsovo bridge.

The Bayesian tools for identifying FE models as well as performing robust prediction analyses are Laplace methods of asymptotic approximation and stochastic simulation algorithms. These tools require a moderate to very large number of repeated system analyses to be performed over the space of uncertain parameters. Consequently, the computational demands depend highly on the number of system analyses and the time required for performing a system analysis. In this work the Transitional MCMC (TMCMC) stochastic simulation algorithm [5] is employed. High performance computing techniques are integrated within the TMCMC tool to efficiently handle large number of DOF in FE models. Fast and accurate component mode synthesis techniques [2] are used, consistent with the FE model parameterization, to achieve drastic reductions in computational effort. Further computational savings are achieved by adopting parallel computing algorithms to efficiently distribute the computations in available multi-core CPUs [3].

2 DESCRIPTION OF BRIDGE AND INSTRUMENTATION

The ravine bridge of Metsovo (Anthohori-Anilio tunnel), shown in Figure 1, of Egnatia Motorway is crossing the deep ravine of Metsovitikos river, 150m over the riverbed. This is the highest bridge of the Egnatia Motorway, with the height of the tallest pier equal to 110m. The total length of the bridge is 537m. The bridge has 4 spans of length 44.78m, 117.87m, 235m, 140m and 3 piers of which M1(45m) supports the boxbeam superstructure through pot bearings (movable in both horizontal directions), while M2(110m) and M3(35m) piers connect monolithically to the structure.

Acceleration measurements were collected under normal operating conditions of the bridge in order to identify the modal properties of the structure (natural frequencies, mode shapes, damping ratios) which constitutes the first step of FE model calibration. The measured data were collected using 5 triaxial and 3 uniaxial accelerometers paired with a 24-bit data logging system and an internal SD flash-card for data storage. The synchronization of the sensors was achieved by using a GPS module in each of the sensors, hence they could be programmed to start and stop recording at exactly the same time for the desired duration. An important aspect of the measurement system is the fact that it is wireless, since this allowed for multiple sets of repeated measurements which is required for accurate mode shape identification. The excitation of the bridge during the measurements was primarily due to road traffic, which ranged from motorcycles to heavy trucks, and environmental excitation such as wind loading and ground micro-tremor, which classifies this case as output-only or ambient modal identification.

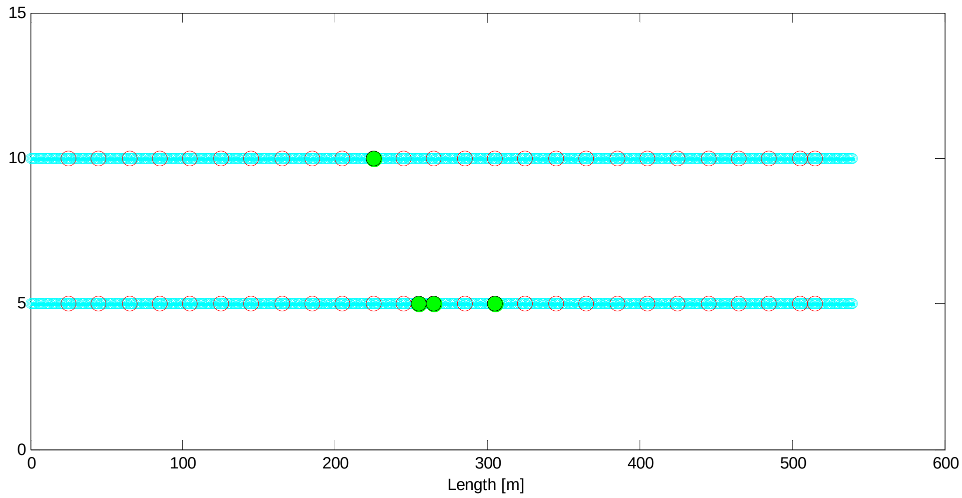

Figure 2: Positions of sensors along deck; Moving sensors (red circles) and reference sensors (green circles)

Given the limited number of sensors and the large length of the deck, multiple sets of measurements had to be performed in order to identify the higher, more complex mode shapes accurately. Specifically, the entire length of the deck was covered in 13 configurations with sensors located approximately 20m apart as shown in Figure 2. Each configuration was recording for 20 minutes with a sampling rate of 100 Hz, and then remained idle for 10 minutes to allow time to move the sensors in the next position. One tri-axial and three uniaxial sensors (one vertical and two horizontal), one at each side of the bridge, remained at the same position throughout the whole measurement process. These sensors are indicated by the filled green circles in Figure 2, and are called the reference sensors. Their purpose is to provide common measurement points along different configurations so as to enable the procedure of mode shape assembling [6]. The locations of the reference sensors, and the distance between successive configurations (20m) were selected based on an optimal sensor location theory [7] so as to provide the highest information content for identifying the modal parameters of the structure. For optimizing the location of the reference sensors, the information entropy [7] was minimized as a function of the location of the reference sensors.

The mode shape components identified in each sensor configuration are differently scaled, and these components were combined into the full mode shape at all measured DOFs of the structure. A mode shape assembly methodology, similar to the one in [6] was followed to resolve this issue of different scaling and produce the desired full mode shape from the local mode shapes taking advantage of the above mentioned optimally placed reference sensors. This is achieved by minimizing the difference between the full mode shape and each of the configuration mode shapes in a least squares sense.

3 ESTIMATION OF MODAL PROPERTIES OF METSOVO BRIDGE

For the extraction of modal properties from the ambient acceleration data, a Bayesian methodology [1] was followed based on the FFT (Fast Fourier Transform) of the acceleration time histories in a specific, user-defined frequency band of interest. This user-defined frequency band must include the resonant frequency and be wide enough so as to comprise all the dynamics associated with that mode, but not so wide so as to introduce unnecessary modeling error. For the definition of an appropriate frequency band, the Power Spectral Densities (PSD) of the acceleration time histories were used along with their Singular Value Spectrums (SVD). Note that the identification methodology does not use these quantities, only the FFT is used in a specific band of interest. The PSD and SVD were used for finding the appropriate bands where resonant frequencies are most likely to exist, and to qualitatively check the recorded data between different sensors. The SVD plot has also the ability to reveal closely spaced modes that are not apparent in the power spectrum, such is the case in the second and third modes of the Metsovo Bridge. The number of possible modes in a frequency band is of importance to the identification algorithm. The PSD and SVD plots revealing the first 3 natural frequencies are presented in Figure 3. Note that in the SVD plot there are two peaks near 0.6 Hz, indicating the existence of two closely spaced modes.

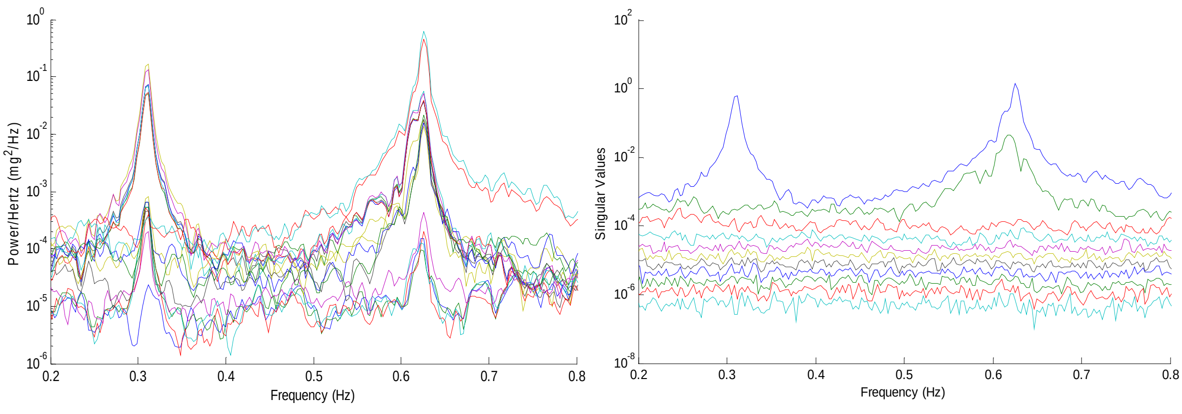

Figure 3: PSD and SVD plots revealing the first 3 lowest modal frequencies.

Following the Bayesian identification methodology and the mode shape assembling algorithm, the natural frequencies and damping ratios of the structure were extracted, and the mode shape components of each configuration were combined to produce the full mode shapes of the structure at all or part of the sensor locations covered by the 13 configurations. For comparison purposes, Table 1 presents the mean and the standard deviation of the experimentally identified modal frequencies for the lowest 10 modes of the Metsovo bridge. Representative assembled mode shapes are shown in Figure 4 and compared with the mode shapes predicted by the nominal fixed-base finite element model of the bridge.

Table 1: Mean and standard deviation (STD) of experimental (Exp.), nominal and model predicted modal frequencies (Hz).

Mode	Type	Exp. Mean	Exp. STD	Nominal	Model Mean	Model STD
1	Transverse	0,3063	0,0007	0,3180	0,3076	0,0049
2	Transverse	0,6034	0,0014	0,6220	0,5960	0,0071
3	Bending	0,6227	0,0008	0,6460	0,6236	0,0081
4	Transverse	0,9646	0,0084	0,9890	0,9227	0,0276
5	Bending	1,0468	0,0066	1,1120	1,1095	0,0107
6	Transverse	1,1389	0,0049	1,1730	1,1381	0,0117
7	Bending	1,4280	0,0042	1,5160	1,4664	0,0167
8	Transverse	1,6967	0,0112	1,7110	1,6444	0,0172
9	Bending	2,0053	0,0054	1,9340	1,8322	0,0203
10	Transverse	2,3666	0,0025	2,3520	2,2192	0,0252

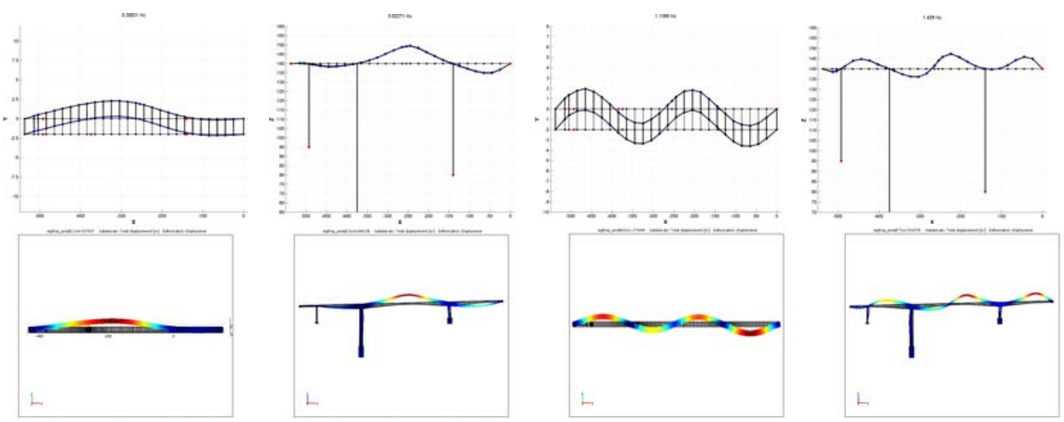

Figure 4: Identified and model predicted mode shapes: *left to right*: first transverse, first bending, fourth transverse, third bending.

4 UNCERTAINTY CALIBRATION OF METSOVO BRIDGE

A detailed FE model of the bridge is created using 3-dimensional tetrahedron quadratic Lagrange finite elements. An extra coarse mesh is chosen to predict the lowest 20 modal frequencies and mode shapes of the bridge. The FE model has 97,636 FEs and 562,101 DOFs. Two model classes of the bridge are considered, a fixed base model and one that models the soil stiffness with translational springs attached in the two horizontal and the vertical directions at each base of the three piers and the two abutments. The FE model is parameterized using five parameters associated with the modulus of elasticity of one or more structural components. For the model with the springs modeling the soil, extra parameters are the soil stiffness which can be considered to be independent for each pier and abutment and for each direction. The model parameters scale the nominal values of the properties that they model.

Model reduction is used to reduce the model and thus the computational effort to manageable levels. Specifically, a parameterization-consistent component mode synthesis (CMS) technique is applied. Details can be found in reference [2]. The reduced model resulting in a number of 406 generalized coordinates, with errors in the estimates for the lowest 20 modal frequencies to be less than 0.02%. Thus, using CMS a drastic reduction in the number of generalized coordinates is obtained which can exceed three orders of magnitude, without sacrificing in accuracy with which the lowest model frequencies are computed. The time to solution for one run of the reduced model is of the order of a few seconds which should be compared to the 2 minutes required for solving the unreduced FE model. Also, in order to further reduce the time to solution, the computations were performed in parallel using 24 cores available from three 4-core double threaded computers.

The prior distribution was assumed to be uniform with bounds in the domain [0.2,2] x … x [0.2,2] for the structural model parameters and in the domain [0.001,1] x [0.001,1] for the prediction error parameters σ_{ω} and σ_{ϕ}. The calibration is done using the lowest 10 modal frequencies and mode shapes identified for the structure. Representative results are obtained in this paper using TMCMC for the fixed-base FE model with five structural model parameters. The TMCMC is used to generate samples from the posterior PDF of the structural model and prediction error parameters and then the uncertainty is propagated to estimate the uncertainty in the modal frequencies of the bridge. The mean and the standard deviation of the uncertainty in the first 10 modal frequencies are presented in Table 1. It can be seen that predictions of the uncertainty for the first 10 modal frequencies are overall closer to the experimental data than the ones predicted from the nominal model. The overall fit between the experimental and the model predicted modal characteristics is summarized in Figure 5 which shows the frequency fits and the mode shape fits using the MAC values.

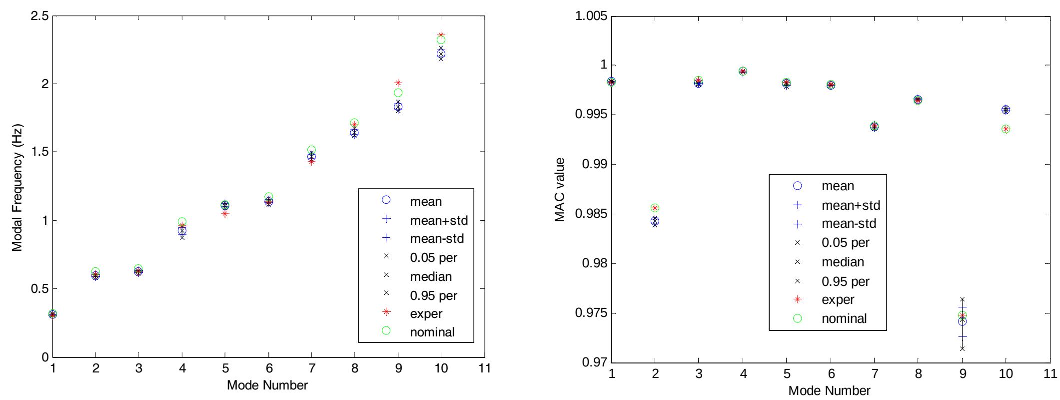

Figure 5: *Left:* Frequency fits, *Right:* MAC values between measured and model predicted mode shapes.

CONCLUSIONS

The proposed Bayesian methodology for FE model parameter estimation using modal properties is flexible to take into account the uncertainties in the modal estimates computed by recently developed Bayesian techniques as well as the different number of mode shape components used for each mode identified by assembling the mode shape from a number of different sensor configurations. Model non-intrusive stochastic simulation algorithms used in Bayesian tools for model uncertainty quantification and calibration, model selection and propagation requires a moderate to large number of FE model simulation runs. For large order FE models with hundred of thousands or even million DOFs and localized nonlinearities encountered in structural dynamics, the computational demands involved may be excessive, especially when a model simulation takes several minutes, hours or even days to complete. Drastic reductions in the time to solution are achieved by integrating model reduction techniques to substantially reduce the order of high fidelity large order FE models, and parallelization techniques to efficiently distribute the computations in available multi-core CPUs. Application of the Bayesian method to Metsovo bridge demonstrated its computational efficiency and accuracy. Specifically, using model reduction techniques and parallelization strategies within TMCMC with the available computing resources, the computational effort was drastically reduced to manageable levels for the high fidelity high-order FE model of the bridge of hundreds of thousands of DOF. In particular, it is demonstrated that the proposed CMS method is efficient in drastically reducing the model to a few hundred degrees of freedom without compromising the accuracy of the results. The model reduction techniques and parallel implementation strategies allowed for two to three orders of magnitude reduction of computational time.

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